Editor's Note: This is the third in a set of articles about ways in which teachers apply the constructivist framework described by author Paul Vermette (2009) in his book, *ENGAGING teens in their own learning: 8 Keys to Student Success.*

Abstract

Using Vermette’s (2009) “ENGAGING framework,” the authors describe their own work in applying the framework in a secondary mathematics classroom. They describe a lesson on linear modeling to help students compare the billing practices of two different hypothetical cell phone companies. The authors focus on Vermette’s eight factors in the ENGAGING framework to demonstrate ways in which the constructivist practices in the lesson align with the framework.

Introduction

In this, the final article of this series, we present a secondary mathematics lesson as a means of demonstrating how the ENGAGING framework developed by Vermette (2009) can operate in secondary classrooms. By systematically examining the cognitive and affective instructional strategies that make this lesson effective, we seek to use the ENGAGING framework to provide a common conceptual language of constructivist based instruction.

In his book, *ENGAGING teens in their own learning: 8 Keys to Student Success,* Dr. Paul J. Vermette sets forth a framework meant to provide teachers with a coherent structure for providing heterogeneous student populations with powerful classroom learning experiences. Vermette synthesizes years of educational research into eight
fundamental components (the ENGAGING framework) that can be used to promote student motivation and achievement in the classroom. These components include:

- Entice effort and build community
- Negotiate meaning
- Group collaboratively
- Active learning and authentic assessment
- Graphic organizers
- Intelligence interventions
- Note-making
- Grade wisely

The following lesson is an exemplar of how effective mathematics instruction can be grounded in the ENGAGING factors. The lesson plan is described using the ‘two step’ outline for effective constructivist teaching first published by Flynn, Mesibov, Vermette, and Smith (2004) and also represented in the ENGAGING framework published by Vermette (2009). Using this design, the lesson is described in terms of its “Exploratory” elements (i.e., the introductory parts of the lesson in which students are engaged with new and possibly confusing aspects of the lesson) and its “Discovery” elements (in which students construct meaning through the use of the ENGAGING components). To help facilitate understanding of how the ENGAGING framework “plays out” in secondary classrooms, the elements of the ENGAGING framework are underlined as they are explored in the context of this mathematics lesson.

**The Cell-phone Lesson**

This lesson is an introductory lesson on linear modeling. By providing a context for studying linear equations, (namely to get the best value for one’s money) this lesson is designed to motivate students to analyze and explore the concept of solving linear equations. For this specific lesson, the student learning target states: “I can use linear
modeling to determine the best value for my money.” In New York State, the appropriate learning standards, strands, and performance indicators for the lesson are as follows:

**Students will recognize and apply mathematics in contexts outside of mathematics.**
8.CN.9 Recognize and apply mathematics to other disciplines, areas of interest, and societal issues

**Students will use the language of mathematics to express mathematical ideas precisely.**
8.CM.10 Use appropriate language, representations, and terminology when describing objects, relationships, mathematical solutions, and rationale

**Students will communicate their mathematical thinking coherently and clearly to peers, teach, and others.**
8.CM.4 Share organized mathematical ideas through the manipulation of objects, numerical tables, drawings, pictures, charts, graphs, tables, diagrams, models and symbols in written and verbal form

**Exploratory (introductory) elements of the lesson.** The lesson begins as students consider nine real-life statistics about the cost, usage and popularity of cell phones (as shown below). With members of their **three person student teams**, their task is to circle two statements they find surprising or interesting and have a brief discussion about why this statistic is true. Students **make notes** as they deem appropriate, and are informed that they will have to publically share their ideas at the end of the allotted time.

- 72% of the U.S. population own cell phones.
- A 2004 MIT survey said that the cell phone was ranked as the one invention that people hate the most, but can’t live without. It beat out the alarm clock and the television!
- The average user replaces his or her cell phone every 18 months.
- Cell phone ownership among 12 to 14 year olds increased from 13 percent in February 2002 to now more than 50%.
- A Let’s Talk (retail company) survey said that 38% of people thought it was okay to use a cell phone in the bathroom.
- More than 36% of all 11- to 14-year-olds own their own cell phone.
- The average American makes approximately 150 cell phone calls per month and leaves 14 voice mails.
- A 2005 University of Michigan study states that 83% of people said cell phones made their lives life easier (choosing it over the internet).

(Wirefly (2009))
Building on the momentum and enthusiasm generated from the whole group discussion, students reflect on the impact that increased cell phone popularity has on both the number of wireless providers and the competition amongst them. While it is logical to assume that students will recognize that as more people want cell phones, the number of cell phone providers will increase, the more intriguing question is “What positive and negative effects does competition among cell phone providers have on American consumers?” Given the fact that the average American has access to many different local, regional, and nationwide providers of cell-phone usage, students are asked to utilize a t-bar graphic organizer to record possible positive and negative impacts of cell phone company competition (in a structure like the one found below).

<table>
<thead>
<tr>
<th>Positive Effects</th>
<th>Negative Effects</th>
</tr>
</thead>
</table>

This discussion leads to the obvious consideration of which provider offers the best value for one’s money. By the end of this learning experience, students will be able to develop a mathematical strategy to determine how to get the best value for their money.
The student teams are then asked to consider the following two hypothetical advertisements:

**General Exchange**
Get exactly what you want and expect from your wireless provider. We offer all the latest phones and cell phone accessories, all with great service. Our efficient service is only $18 a month plus $0.15 per call.

**National Wireless**
Enjoy convenience, simplicity and savings with National Wireless. We offer excellent service anywhere you travel. Our quality service is $24 a month plus $0.10 per call.

Based on their initial reaction to the ads, student teams predict which of these cell phone providers is offering the best deal. Provided with the following graphic organizer to record their prediction, students are encouraged to use relevant mathematical evidence to support their claim.

**Our Prediction:**
We think that ____________________________
will be the best deal because ____________________________
_____________________________________________
_____________________________________________

**Rationale (Evidence):**
1.)
2.)
3.)

Since this learning experience has been designed for and conducted with ninth graders, we stress the fact that the opportunity to make a written prediction builds motivation for the learning experience by enabling students to determine the correctness of their initial
judgment. In addition, since these predictions must be grounded in mathematical reasoning, students are utilizing their intuitive notions of linear modeling before any formal explanations are given. By “playing” with the mathematical reasoning first, the teacher’s role in this learning experience is to expand on the ideas already presented by the learners, providing students with the common mathematical language used to describe their ideas.

**Discovery (meaning construction) elements of the lesson.** After student teams have drafted a prediction as to which company provides the cheapest cell phone service, each team shares their ideas and mathematical rationale as part of a whole group discussion. As students provide examples of costs with each company, the teacher records their ideas in two charts similar to the format shown below. After the teacher has the necessary information recorded, students are given the opportunity to “make notes” from the information provided by their classmates. Since at point in the lesson students begin to make meaning from the data presented, students must determine how to organize variables such as initial cost, cost per minute and number of calls, to determine which cell phone company is offering the best deals.

<table>
<thead>
<tr>
<th></th>
<th>General Exchange</th>
<th>National Wireless</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Calls</td>
<td>Cost per Month</td>
<td>Number of Calls</td>
</tr>
<tr>
<td>25 calls</td>
<td>$21.75</td>
<td>25 calls</td>
</tr>
<tr>
<td>110 calls</td>
<td>$34.50</td>
<td>110 calls</td>
</tr>
<tr>
<td>300 calls</td>
<td>$63.00</td>
<td>300 calls</td>
</tr>
</tbody>
</table>
The “General Exchange” table above presents a mathematically correct, yet paradoxical situation. In some cases, “General Exchange” is the less expensive cell phone provider, yet in other cases, it is more expensive than its competitor is. How could this be the case? Students explore this idea with the following four questions that require them to model, reflect and expand upon this situation:

a. How does the monthly charge for 100 calls with General Exchange compare with the monthly charge for 100 calls placed with National Wireless?

b. Given that you have $40 a month to spend on your cell phone bill, how many calls could you make in a month for $40 if you have General Exchange? How many could you make with National Wireless?

c. For what number of calls is General Exchange cheaper? For what number of calls is National Wireless cheaper? How do you know?

d. Which plan would be best for you or your family?

Reflection question (d) addresses the learning target of this lesson and is the item that students need to think deeply about as they perform mathematical calculations. Since our goal is to help students use linear modeling to develop a strategy for determining the best value, one authentic assessment option for demonstration of this knowledge is to write a letter to a parent describing the best plan for them (or their family). With the help of their teams, learners will brainstorm what sort of information is necessary to provide evidence for the cheapest wireless provider. Student responses are likely to include but not limited to:

- Information regarding the frequency in which he/she uses a cell phone and the impact that has on choosing the cheapest cell phone provider
- Specific mathematical calculations supporting why that company is a better buy,
- Information regarding how he/she came to this conclusion

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Students are then to collaborate with their teammates to create an outline of their letters. Although this outline is created together, by the end of the lesson each student must write an individual letter so the teacher can assess personal understanding.

Students complete the rough draft in class, so the teacher can clarify and address any mathematical misconceptions before students write their final copy for homework that night. An assessment rubric for the letter focuses on student learning and student thinking:

**Letter Rubric**

<table>
<thead>
<tr>
<th>Organization: The purpose of the letter and its discussion help the reader understand the letter’s message</th>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very well organized letter providing a logical rationale and a clear pathway to its conclusions.</td>
<td>Well organized letter although the rationale is not fully developed.</td>
<td>Organization makes it somewhat difficult for the reader follow the writer’s logic</td>
<td>Lack of organization makes it very difficult to follow the writer’s logic</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Information about your own cell-phone usage</th>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very clear and well-presented providing good reason for consideration.</td>
<td>Clear and correctly presented.</td>
<td>Somewhat difficult to understand.</td>
<td>Lacking coherent information about cell-phone usage.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Demonstration of mathematics understandings</th>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematical evidence displayed is both conceptually and computationally accurate.</td>
<td>Mathematical evidence displayed is conceptually accurate but contains several minor computational errors.</td>
<td>Mathematical evidence displayed is computationally accurate but contains several conceptual errors.</td>
<td>Mathematical evidence displayed contains both conceptual and computational errors.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Appearance of the letter created</th>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Neat, easy to read, and mechanically correct.</td>
<td>Neat and readable, but containing minor mechanical errors.</td>
<td>Relatively neat and readable, but containing significant mechanical errors.</td>
<td>Very weak due to problems with neatness, readability, and mechanics.</td>
<td></td>
</tr>
</tbody>
</table>

An alternative assessment option is to have students compare his or her family’s current cell phone contract to the fictitious contract of General Exchange customers (as
provided by the teacher). By extending students’ understanding to their own families’ cell phone contracts, students will create a poster comparing and contrasting the two agreements. Like the letter, a plan for creating the poster should be started in class so the teacher can provide support as needed and can be finished at home if required. Both assessment options allow students to provide evidence of their understanding of the learning target of using linear modeling to explain the “best buy.” The assessment rubric for this alternative assignment focuses on student learning and student thinking:

**Poster Rubric**

<table>
<thead>
<tr>
<th>Similarities between your cell phone contract and the General Exchange cell phone company contract</th>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>The poster describes in detail three ways General Exchange cell phone company contract is <em>like</em> your cell phone contract.</td>
<td>The poster states three ways General Exchange cell phone company contract is <em>like</em> your cell phone contract but the similarities are not explained in detail.</td>
<td>The poster states one or two ways General Exchange cell phone company contract is <em>like</em> your cell phone contract but does describe the similarities in detail</td>
<td>The poster does not state how ways General Exchange cell phone company contract is <em>like</em> your cell phone contract.</td>
<td></td>
</tr>
</tbody>
</table>

| Differences between your cell phone contract and the General Exchange cell phone company contract | The poster describes in detail 3 ways the General Exchange contract is *different* from your cell phone contract. | The poster states 3 ways the General Exchange contract is *different* from your cell phone contract but not in detail. | The poster states 1 or 2 ways General Exchange contract is *different* from your contract, but not in detail. | The poster does not state how the General Exchange contract is *different* your cell phone contract. |

| Demonstration of mathematics understandings | Mathematical evidence displayed is both conceptually and computationally accurate. | Mathematical evidence displayed is conceptually accurate but contains several minor computational errors. | Mathematical evidence displayed is computationally accurate but contains several conceptual errors. | Mathematical evidence displayed contains both conceptual and computational errors. |

| Appearance of the poster created | The poster is neat, colorful and contains images that enhance your final product. | The poster is neat and colorful but does not contain images that enhance your final product. | The poster is either neat or colorful but does not contain any images to enhance your final product. | The poster is not neat or colorful and does not contain images that enhance your final product. |
Analysis of the lesson using the ENGAGING Framework

In this section, we will provide our commentary regarding how these steps of the cell phone lesson fit into all of the components of ENGAGING model. To review, the factors of the ENGAGING framework are:

- Entice effort and build community
- Negotiate meaning
- Group collaboratively
- Active learning and authentic assessment
- Graphic organizers
- Intelligence interventions
- Note-making
- Grade wisely

**Entice effort and build community:** How does the teacher entice excellence in effort?

With the exception of the final letter writing or poster creation, virtually all of the learning in this lesson takes place in cooperative learning settings. With their groups, students examine cell phone statistics, create the graphic organizers, develop linear equations, answer discussion questions and construct a deeper understanding of the concept of linear equations as they discuss and explore these issues with their peers. In a lesson where ENGAGING learning takes place, students must articulate their ideas and developing understandings with others. It is through the communication of ideas that increased transfer of essential mathematical and affective understandings takes place (Jones, Jones & Vermette, 2009a; Jones, Jones & Vermette, 2009b)

Though not specified in the above description, we assume that the ENGAGING practitioner will be ‘working the room’ as students perform all the activities highlighted as key parts of the learning experience. As described by Vermette (1998) and
Konkoski-Bates & Vermette (2004), effective teachers monitor, encourage and provide interventions as appropriate when they supervise student teams by means of an informal walk through.

**Negotiate meaning** (How do the students make their own meaning?)

In an effort to deconstruct this ENGAGING factor, consider the portion of the lesson in which students examine cell phone statistics and record their ideas about the positive and negative effects of cell phone company competition in a T-bar graphic organizer. In a less ENGAGING classroom, the teacher might have just told the students that competition forces cell phone companies to provide customers with different service plans. Instead, allowing students to come to these conclusions for themselves sets up the rationale for determining the best buy. Such an activity gives students the opportunity to invest themselves in the subsequent learning experiences and provides them with a forum for documenting intuitive mathematical notions they may later adapt and revise.

Cell phones are a large part of contemporary America and current events, and so by having students use mathematics in order to understand everyday phenomena, they are motivated to play with these concepts and problem-solve to develop solutions. Planning with the ENGAGING framework means that educators realize that they cannot do the learning for their students. Rather it is the teacher’s job to create learning experiences where students make meaningful connections between ideas, apply their current understandings to new situations (transfer), and extend their knowledge to other facets
of their own lives. In an ENGAGING and constructivist classroom, the student must do the thinking in order for meaningful learning to take place.

**Group collaboratively: How do the students collaborate with each other?**

Cooperative learning is heavily utilized throughout this lesson and functions as both an instructional strategy and an inherent source of student motivation. This learning experience is intentionally designed so that most tasks are completed with others - thereby provoking each student to strive for excellence, if not for himself or herself, then for the collective good of the team. By allowing students the opportunity to work in these cooperative learning settings, one improves students’ cognitive mathematical abilities while allowing for the opportunity to practice affective skills such as collaboration and cooperation. It is the deliberate practice of interpersonal and intrapersonal skills that not only increases affective competencies, but increases academic achievement (Jones, Jones & Vermette, 2009c).

**Active learning and authentic assessment: What active learning activities are utilized?**

The authentic assessment options available to teachers in this lesson are 1) a letter to parents or 2) a poster comparing the students’ own cell phone contract to the one provided from General Exchange. The writing assignment is a valuable assessment of student understanding because it allows learners to articulate their learning fully on a topic of high interest for students at the secondary level. The poster is authentic in that is created from the students’ actual cell phone contract and thus is that it is directly created from elements of the students’ real life. Because both assessment options
assess the same learning target, we recommend allowing students to make the decision of how to demonstrate their understanding, thus increasing student motivation and ownership in the assessment.

**Graphic organizers: How do the learners utilize graphic organizers to structure their understandings?**

This lesson includes several graphic organizers, designed specifically to help students record and interpret the costs for the cell phone providers. Especially at the secondary level, where clear and meaningful articulation of understanding is essential, graphic organizers are an asset in facilitating this process. Furthermore, we suggest a discussion regarding strategies for organizing cell phone data would be a very valuable discussion to have with a classroom of learners. In this way, students think deeply about not only the structure of their notes but also what other documentation options look like. It is this internalization of writing and organizing structures which can be immediately transferred to other academic areas and other facets of students’ lives.

**Intelligence interventions: How do the learners respond to the teacher’s queries and what role does multiple intelligences come to play in their thinking?**

In ENGAGING classrooms, teachers must not only use multiple intelligences, but they must maximize their potential by using them correctly. In an ideal situation, students should first work within their preferred intelligence as they are introduced to a mathematical topic. This will allow reluctant mathematics students to feel more comfortable while providing them with the motivation to continue with the learning
process. Then once students have mastered the skill, working outside of one’s preferred intelligence will allow students to think abstractly and critically about the material as they gain a deeper understanding of linear modeling. In this cell-phone lesson, the multiple intelligences were employed in the following ways:

- Students worked in three person teams to read and discuss the cell phone statistics that are most interesting to them. *(Intrapersonal Intelligence)*

- Students recorded the positive and negative effects of cell phone competition in a T-Bar graphic organizer. *(Visual-Spatial Intelligence)*

- Students compared two cell phone ads and recorded their predictions as to which they one is the better deal. *(Verbal-Linguistic Intelligence)*

- Students modeled, reflected and expanded upon their predictions by developing mathematical evidence to support their claim using linear equations. *(Logical-Mathematical Intelligence)*

- Students discussed and recorded their thoughts on several discussion questions using the information they just discovered. *(Verbal-Linguistic Intelligence and Logical-Mathematical Intelligence)*

- Students individually wrote letters or created posters explaining which provider has the better cell phone plan for them and why. *(Verbal-Linguistic Intelligence and Intrapersonal Intelligence)*

It is not a coincidence that the verbal-linguistic intelligence is very prevalent in the beginning of this lesson. Since many students are highly developed with their verbal-linguistic and interpersonal skills, but lacking in their logical-mathematical abilities, the cooperative learning inherent in this lesson allows for the teacher to build on student strengths as they work through more abstract mathematical procedures. Obviously, the logical-mathematical intelligence is woven throughout this lesson, but using additional intelligence interventions diversifies and strengthens the lesson.
Note-making: How did the students document their developing ideas?

The students documented their developing ideas through note-making in the form of graphic organizers, recording predictions, and letter writing or poster creation. Note that though these formats, students make notes, rather than take notes. Because all the documentation is student-constructed, it is likely personally relevant and meaningful. Students are actively learning through this documentation process since they must synthesize and interpret the mathematical concepts being presented while summarizing and reflecting on their learning experiences. While no two students will have the same notes at the end of this lesson, their notes will be an asset to the individual’s overall understanding.

Grade wisely: What role does grading play in the learners’ efforts?

Being that students must demonstrate their understanding in this lesson in an authentic and meaningful way, grading student work judiciously is essential. Whether one chooses to have students write letters home or create a poster comparing cell phone contracts, grading should be a source of motivation for continued student achievement. In both cases, a rubric is an essential component to grading wisely. Whether this rubric is co-created with students or provided by the teacher, it provides all with a clear standard of quality.

Conclusion

In this article, a secondary mathematics lesson on linear modeling was used to examine how Vermette’s (2009) ENGAGING model can be applied in a constructivist classroom setting. All eight components of the ENGAGING model were described and analyzed in
an effort to help readers recognize the ways in which the model can be applied.

Clearly, the model can be useful in other subject areas than mathematics, and we encourage teachers to develop ways to apply it in their own subject fields.

References Cited


Jones, K., Jones, J., & Vermette, P. (2009a) Using intentional transfer techniques to promote the cognitive and affective growth of middle level learners. In the Middle 28(1), 9-13.

